The Asian Option and Spread Option Evaluation for S&P500 Index and NASDAQ Index

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Abstract: During the COVID 19 pandemic, investors have been suffering from huge risks and losses. In this case, it is necessary to have different types of financial products for them to hedge the risk. The purpose of this study is evaluating spread option and Asian option for S&P500 Index and NASDAQ Index. Lognormal pricing model method and Monte Carlo method are used in this study for option pricing and simulation. Data was obtained from Yahoo Finance and data processing as well as simulations were carried out based on Microsoft Excel. According to data analyze, the following results showed up. By buying both assets in spread option, investors would be able to hedge their risks. Besides, Asia option are cheaper than regular option and may have less risk. However, certain limitations like small sample size (only S&P500 and NASDAQ-100 were taken into consideration) of this study should be noticed. These results shed light on comparation of spread option and Asian option evaluation.

1. Introduction

Option has become a popular financial product for many years, which is the right to buy or sell on a future date (maturity) at a price (future price) determined today, separating ownership from risk. Different from futures that imposes obligations to buy or sell in the future on investors, options offer them the right not to buy or sell in the future. Based on this product, some derivatives (e.g., exotic option) are developed to meet investors' needs. Financial option on securities and futures contracts trade on exchanges. Options trade on equities, bond futures, interest rate futures, commodity & currency futures. Among all kinds of options, we are going to talk about Spread Options and Asian Options in this study.

Contemporarily, spread option is appearing more and more frequently in the market. Because of the rules of spread option, which takes two or more assets to analyze the option pricing, it allows traders to take their spread. It could be expressed in many financial products as well, e.g., equities, bonds, currencies, etc. For example, the crush spread option is always used in soybean market by analyzing the difference between the value of soybean or the combination of products (soybean meal & soybean oil) in CNOT.

Asian option is one of the exotic options, whose payoff depends on the average price over a certain period. Based on this characteristic, Asian option holds less market manipulation risk. Zheng and Chen (2000) claim that Asian option avoids the volatility risk caused by frequent transactions because of its path-dependent characteristic [1]. In 1987, an option of crude oil with pricing by the average price was developed by Standish and Spaughton in Tokyo firstly. Asian option is a popular choice for hedging because of its lower risk and price. Some researches show the feasibility and reliability of this case, e.g., Yang and Zou (1999) provide a hedging strategy of Asian option by using the generalized Clark-Ocone formula [2].

Option has always been getting a wide range of attention from the financial world, therefore, it's not hard to find out that many researches have discussed about options, including pricing, sensitivity analyzing, etc. [3-7].

The rest part of the paper is organized as follow. The Sec. 2 will introduce the data origination as well as processing and simulation methods. Subsequently, the Sec. 3 will demonstrate data processing and simulation results for both Asian option and spread option and a discussion of comparison of these two options and limitations of this research. Afterward, the Sec. 4 will give a brief summary.

2. Data & Method

2.1 Data

To price the Asian option, the S&P 500 daily adjusted prices are selected for analysis from 2020.9.1-2021.8.31 As for spread options, NASDAQ-100 data is also collected beside the data of the S&P with the same time period to make the spread option pricing model. The price trends of the two underlying assets are shown in Fig. 1, These data are searched from Yahoo finance. The risk-free interest rate is collected from US department of the treasury.

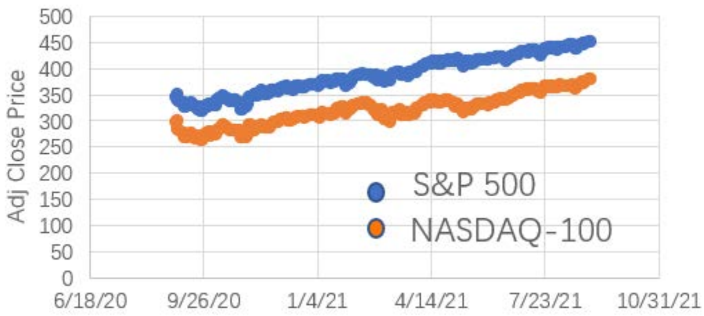

Fig. 1. S&P500 and NASDAQ-100 Daily Adj Close Price From 2020/9/1 to 20218/31.

2.2 Method

In order to price the option, Monte-Carlo (M-C) simulations are utilized based on the random number generator from the Microsoft Excel. M-C Simulation is a research method used for random variables events. Based on a particular statistical model, a number of random samplings are done to get corresponding valuation which can be used to determine the final pricing later. M-C Simulation is appropriate for this pricing research with the following reasons.

M-C Simulation can be implemented easily. With the utilization of software (e.g., Excel), the large number of random samplings and the calculation for average value in M-C Simulation can be achieved easily by specific functions.

Additionally, M-C Simulation is an efficient manner for this research. Li (2007) contends that M-C Simulation has an advantage on solving high-dimensional problem since its convergence rate is independent to the dimension of problem while the convergence rate of many other numerical methods, like trapezoidal rule, would be lower with the growth of the dimension of problem [8]. In other words, it means that M-C Simulation needs less random sampling to convergence than other methods. Lai and Feng (2009) concluded that the M-C Simulation is expected to play an important role in the pricing of high-dimensional derivative securities with multiple assets and path-dependent characteristic because it is simple to implement and independent to the dimension of problem [9].

M-C Simulation considers situations comprehensively. With sufficient random samplings, most of cases can be included in the simulation. As a result, the pricing would be accurate. Raychaudhuri (2011) argue that the M-C Simulation provides a study with global input for researchers. In contrast, a traditional method by testing different visions, such as best case, base case, and worse case, of a model is hardly to consider all circumstances [10].

Specifically, we collected the data in S&P 500 as Asia option required, where the data description is given in Table. I.

Table I: Asia Option Requires Data

r	0.07%
Sigma	12.22%
Spot Price	4,496.19
T	22 days (0.087301587 years)
delta	0
Strike Price	4500

Then we use Monte-Carlo Simulation method to analysis them in excel. For both Spread option and Asian option, we used lognormal pricing model to estimate the future stock price. In the spread option, as it has two stock to estimate, we assume that the prices of Nasdaq-100 and S & P500 are distributed in lognormal. After obtaining the stock price of the next day, we also use the Monte-Carlo simulation method. We used 1000 random numbers to get the price of 1000 stocks by following the lognormal pricing model, which is statistically enough for obtaining a robust result. Particularly, as for spread options, two correlated random numbers are generated to price the two underlying assets (S&P500 and NASDAQ-100). We use the formula Max {S2(t)-S1(t)-k, 0}, Max {k-S2(t)-S1(t),0} of the spread option pricing model to derive the call spread option payoff and put spread option payoff. For the strike price in our analysis, we use the spot price of NASDAQ-100 minus the spot price of S&P500. In this analysis process, we mainly use call options for analysis, hence we calculate the average value of 1000 call options and calculate the present value of call options. Afterward, we use the present value of call spread option and 29 volatile values corresponding to the present value of call spread option to change the volatility. The description of the data is presented in Table. II and the sigma of the S&P500 and NASDAQ -100 as a function of call spread payoff are illustrated in Figs. 2 and 3, respectively.

Table II: S&P 500, Nasdaq-100 Required Data

Asset	S&P500	NASDAQ-100
r	0.07%	0.07%
Sigma	14.84%	21.34%
Spot Price	4522	15582
T	1	1
Delta	1.30%	0.49%
Spread Opt Strike	11060	
Correlation	0.866973771	

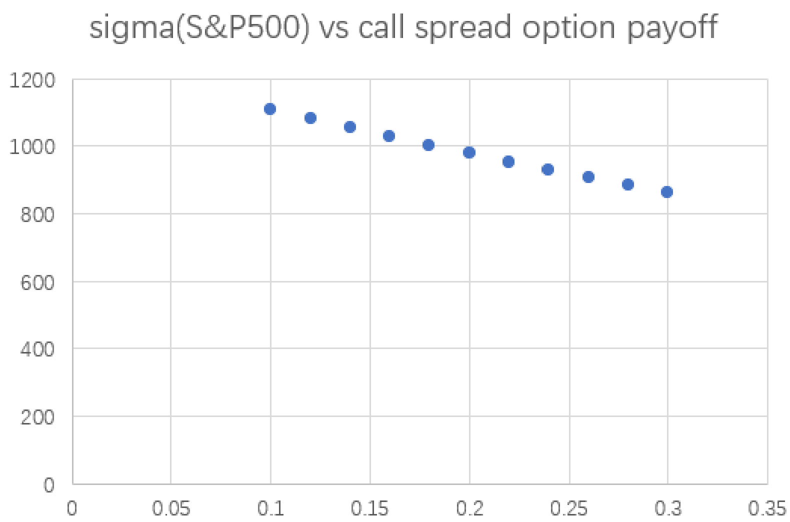

Fig. 2. Sigma (S&P 500) vs Call Spread Option Payoff.

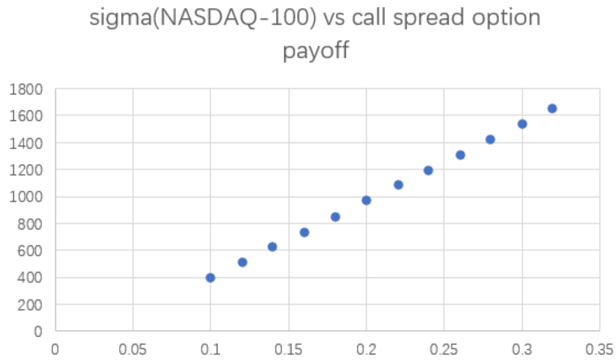

Fig. 3. Sigma (NASDAQ-100) vs Call Spread Option Payoff.

3. Results & Discussions

3.1 Results & Discussions

In order to visualize the data vividly and see the data more intuitively, the results of simulation are converted to tables and figures. The comparisons between Asian option and regular option are depicted in Fig. 4. Afterward, we calculate the average value of call option and put option to obtain the value in maturity time and evaluate the present value and max error. Similarly, the procedures are carried out similarly for regular option.

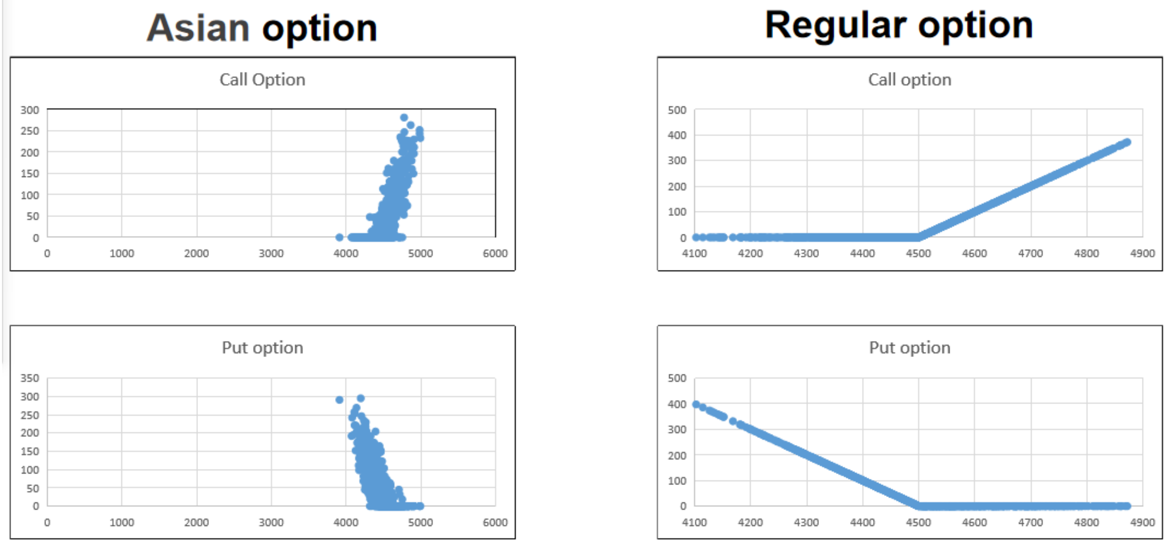

Fig. 4. Comparison with regular option.

TABLE III: Asian options and regular options comparison

	Asian Options		Regular Options	
	Call option	**Put option**	**Call option**	**Put option**
Time T Value	38.024	40.874	62.872	62.816
Time 0 Value	38.022	40.872	62.869	62.812
Max error	3.621	3.686	5.829	5.764

Subsequently, we put the Asian options and regular options data in Table. III and the relationships can be found accordingly. As illustrated in Fig. 5, if the stock price moves down by $1, the Asian put option prices moves up by 0.96dollars. If the stock price moves down by $1, the Asian put option prices moves up by 0.91 dollars. Seen from the Fig. 6, if the stock price moves up by $1, the Asian call option prices moves down by 0.9458 dollars. If the stock price moves up by $1, the Regular call option prices moves down by 0.89 dollars. Based on Fig. 7, if the stock price moves up by 1%, the Asian call option prices moves up by 2.87 dollars. If the stock price moves up by 1%, the Regular call option prices moves up by 4.9 dollars. According to Fig. 8, if the stock price moves up by 1%, the

Asian put option prices moves up by 3.08 dollars. Whereas, if the stock price moves up by 1%, the Regular put option prices moves up by 5.14 dollars.

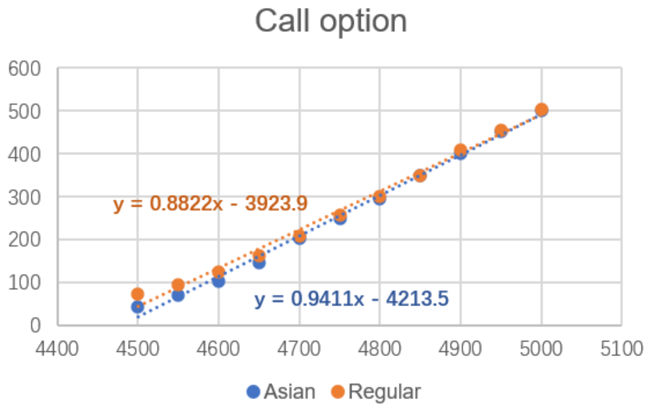

Fig. 5. comparison of spot price in call option of Asian and regular options.

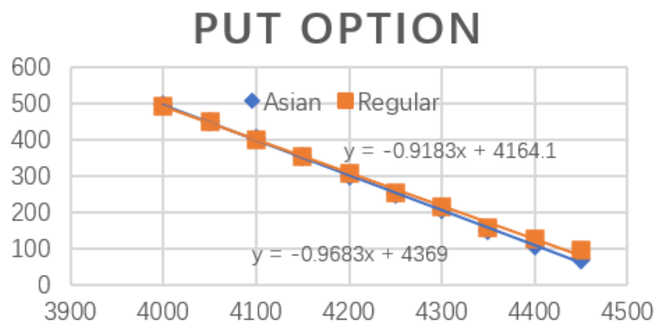

Fig. 6. comparison of spot price in put option of Asian options and regular options.

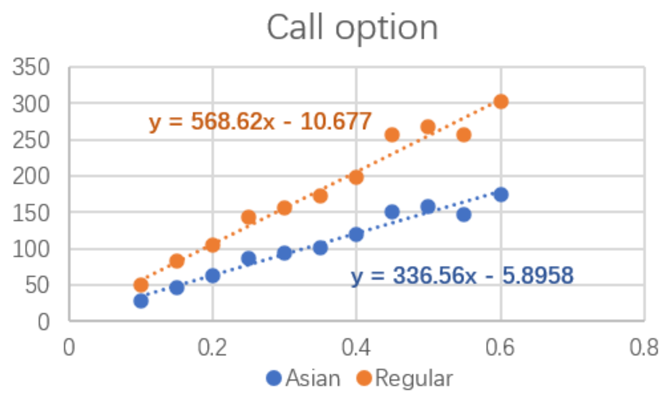

Fig. 7. comparison of sigma in call option of Asian option and regular options

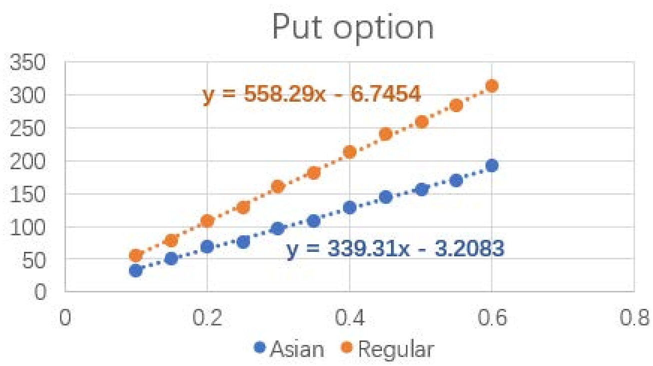

Fig. 8. comparison of sigma in put option of Asian option and regular options

3.2 Spread Options

In the spread option, we determined that St = S2-S1 is greater than strike price K. in the sigma study, because we have two sigma, S & P500 and Nasdaq-100. When the sigma of S & P500 increases, the call spread option payoff decreases. When the sigma of Nasdaq-100 is incremented, the call spread option payoff is incremented. When calculating the option payoff, we set St= Nasdaq-100 (S2) - S & P500 (S1), so when the volatility of Nasdaq-100 is greater, it will be greater relative to the option payoff.

TABLE IV: The Simulation results

S&P500			NASDAQ-100		
Date	Adj Close	Returns	Date	Adj Close	Returns
2020/9/1	347.425262		2020/9/1	298.6533	
2020/9/2	352.450439	0.014464	2020/9/2	301.4813	0.009469
2020/9/3	340.321106	-0.03441	2020/9/3	286.1961	-0.0507
2020/9/4	337.54245	-0.00816	2020/9/4	282.3823	-0.01333
2020/9/8	328.319794	-0.02732	2020/9/8	268.8098	-0.04806
2020/9/9	334.803284	0.019747	2020/9/9	276.7064	0.029376
2020/9/10	328.989868	-0.01736	2020/9/10	271.1898	-0.01994
2020/9/11	329.157349	0.000509	2020/9/11	269.3077	-0.00694
2020/9/14	333.492798	0.013171	2020/9/14	273.9979	0.017415
2020/9/15	335.177704	0.005052	2020/9/15	277.8813	0.014173
2020/9/16	333.847473	-0.00397	2020/9/16	273.4502	-0.01595
2020/9/17	330.911224	-0.0088	2020/9/17	269.1783	-0.01562
2020/9/18	327.101532	-0.01151	2020/9/18	265.7429	-0.01276
2020/9/21	323.461029	-0.01113	2020/9/21	266.3802	0.002398
2020/9/22	326.75531	0.010184	2020/9/22	271.3292	0.018579
2020/9/23	319.177551	-0.02319	2020/9/23	263.0443	-0.03053
2020/9/24	320.02829	0.002665	2020/9/24	264.2691	0.004656
2020/9/25	325.202148	0.016167	2020/9/25	270.4131	0.023249
2020/9/28	330.603577	0.016609	2020/9/28	276.0293	0.020769

Payoff at maturity max(S2(t)-S1(t)-K,0), i.e., the underlying is the difference between two values. The payoff when option expires is asset 2 price(S2) minus asset 1 price(S1) minus the strike price(K). S2 minus S1 is just the spread of two asset where spread is greater than strike price K.

We also carried out Monte Carlo simulations involving estimating multiple results and averaging them, which includes estimating the results of a given procedure by estimating the values of an uncertain variable. The Monte Carlo simulations are applied for the sake of finding out the if changing volatility will affect option payoff trend.

From the Table. IV, one realizes that the volatility of S&P500 is lower than the volatility of NASDAQ-100 according to their sigma. As shown in Figs. 2 and 3, the sensitivity of our spread option price to sigma is different for S&P500 and NASDAQ-100. The price of S&P has a positive correlation to the change of its volatility, while the price of NASADAQ-100 has a negative one. Meanwhile, NASDAQ-100's price has a stronger correlation, or a higher sensitivity to its sigma than S&P500 does.

3.3 Limitation

However, there are some limitations of this research. First of all, we use one year data to analysis. On this basis, the result can be more blurred than using daily ones. In addition, the M-C simulation is not suitable in all scenarios. M-C simulation is only available in normal distribution. In daily life and reality, not all cases are normal distribution.

4. Conclusion

In summary, lognormal pricing model method and Monte Carlo method are utilized to evaluate Asian option and spread options based on the data collected from Yahoo Finance. According to the comparisons of S&P500 and NASDAQ-100, their prices have different volatilities and different sensitivities to their volatilities, which would make a good choice on spread option. Additionally, investors would be able to hedge their risks buying both assets in spread option. What they can pursue here can be “always win or breakeven even when things going wrong. As for Asia options, they are cheaper than regular option and may have less risk. These results offer a guideline for investors choosing suitable option during making decision.

5. Conflict of Interest

The authors declare no conflict of interest.

6. Author Contributions

These authors contributed equally.

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